Rabdos AI: Sample Problems

A collection of verified math problems.

1. Group Theory

Let n beads be arranged on a circle and indexed $0, 1, \ldots, n-1$. Each bead is colored by an element of the finite field $\mathbb{F}_7 = \mathbb{Z}/7\mathbb{Z}$. Two colorings $(c_0, \ldots, c_{n-1}) \in \mathbb{F}_7^n$ are considered equivalent if one can be obtained from the other by a rotation or reflection of the necklace (i.e. by the usual dihedral action D_n on indices).
A coloring is *admissible* if it satisfies all three linear constraints in $\mathbb{F}_7$:

$$\text{(C0)} \sum_{i=0}^{n-1} c_i \equiv 0, \qquad \text{(C1)} \sum_{i=0}^{n-1} i\, c_i \equiv 0, \qquad \text{(C2)} \sum_{\substack{0 \le i \le n-1 \\ i \equiv 0 \pmod 3}} c_i \equiv 1.$$

For this problem, take $n = 30$.

How many D_n–equivalence classes of admissible colorings are there for $n = 30$? Report your answer modulo 1,000,003 as a positive integer.

Answer (mod 1,000,003**):** $\boxed{587{,}104}$

Evaluation: `[Gemini: 0/5]` `[GPT-5-Thinking: 1/5]`

Metadata.

- **Difficulty:** Medium–Hard
- **Subject:** Group Theory – Actions on Colorings
- **Technique:** Burnside's Lemma (with linear constraints over $\mathbb{F}_7$)
- **Reasoning trace.** Burnside reduction over D_{30} to cycle variables $\Rightarrow$ impose the three global linear constraints as a $3 \times d(g)$ system in $\mathbb{F}_7$ per conjugacy type $\Rightarrow$ sum fixed-point counts and divide by $|D_{30}|$.

2. Algebraic Combinatorics

Let H be the set of all hyperplanes in $\mathbb{R}^{48}$ given by $x_i - x_j = 0$ or $x_i - x_j = 1$ for every pair of indices with $1 \le i < j \le 48$. Intersect H with the hyperplane $x_1 + x_2 + \cdots + x_{48} = 0$ to obtain an arrangement inside that 47-dimensional subspace.
Let r be the number of regions of this arrangement, and let b be the number of relatively bounded regions. What is the remainder of $(r - b)$ when divided by 48?

Answer (mod 48**):** $\boxed{2}$

Evaluation: `[Gemini: 1/5]` `[GPT-5-Thinking: 1/5]`

Metadata.

- **Difficulty:** Medium–Hard
- **Subject:** Hyperplane Arrangements

- **Technique:** Identify as Shi arrangement (type A); apply Athanasiadis' formula and Zaslavsky's theorem.
- **Reasoning trace.** Recognize Shi arrangement of type A_{47} $\Rightarrow$ use $\chi_{\text{Shi}}(t) = (t-n)^{n-1}$ with $n = 48 \Rightarrow r = (-1)^{47}\chi(-1) = 49^{47}$, $b = (-1)^{47}\chi(1) = 47^{47} \Rightarrow r - b \equiv 49^{47} - 47^{47} \equiv 1 - (-1) \equiv 2 \pmod{48}$.

3. Probability

Consider a random permutation of the letters in the word `MISSISSIPPI`. Each permutation is equally likely. What is the probability that no two **S**'s stand next to each other, no two **I**'s stand next to each other, and no two **P**'s stand next to each other?

Answer: $\boxed{\frac{16}{275}}$

Evaluation: `[Gemini: 0/5]` `[GPT-5-Thinking: 4/5]`

Metadata.

- **Difficulty:** Medium–Hard
- **Subject:** Probability
- **Technique:** Counting runs.
- **Reasoning trace.** Total number of distinct permutations of `MISSISSIPPI` $= \frac{11!}{4!4!2!} = 34650$ $\Rightarrow$ choose 4 non-adjacent slots for S's among 11 ($\binom{8}{4} = 70$) $\Rightarrow$ from remaining 7 positions, choose 4 non-adjacent for I's $\Rightarrow$ from remaining 3, choose 2 non-adjacent for P's (last slot for M) $\Rightarrow$ total favorable $= 2016 \Rightarrow$ probability $= \frac{2016}{34650} = \frac{16}{275}$.

4. Topology

Let X be the set of real numbers and define

$$A = \left\{ \frac{1}{n} \;\middle|\; n = 1, 2, 3, \ldots \right\}.$$

Define a topology τ on X by declaring that a set $O \subset X$ belongs to τ if and only if

$$O = U - B,$$

where U is an open set in the usual Euclidean topology on $\mathbb{R}$ and $B \subset A$.
Determine whether the topological space (X, τ) is **countably paracompact.** Write `1` for true and `0` for false.

Answer: $\boxed{0}$

Evaluation: `[Gemini: 3/5]` `[GPT-5-Thinking: 1/5]`

Metadata.

- **Difficulty:** Medium
- **Subject:** Point-Set Topology – Separation and Covering Properties

- **Technique:** Construction of a non-locally-finite refinement for a countable open cover. This topology is sometimes called the *Smirnov topology* on X.
- **Reasoning trace.** X is not countably paracompact $\Rightarrow$ the countable open covering by $O_n = X - (A - \{\frac{1}{n}\})$ has no open locally finite refinement $\Rightarrow$ any open set containing 0 must intersect infinitely many others since every neighborhood of 0 includes an interval about 0 minus finitely many $\frac{1}{n}$ points $\Rightarrow$ local finiteness fails for all countable refinements.

5. Epistemic Logic

Consider a multi-agent epistemic model with three agents A, B, C operating in the multi-agent logic $\mathsf{S5}$. The set of possible worlds is $W = \mathbb{F}_2^{12}$, the set of all 12-bit binary strings $x = (x_1, x_2, \ldots, x_{12})$ with arithmetic modulo 2 (XOR). The designated actual world is $w_\star = 0$ (the all-zeros string). Each agent i has an equivalence relation R_i on W representing indistinguishability. We define these via linear subspaces of $\mathbb{F}_2^{12}$:

Agent A: Let $U_A = \operatorname{span}\{e_1, e_2\}$ where e_j is the jth standard basis vector. Define $x\, R_A\, y$ iff $x - y \in U_A$ (equivalently, x and y differ only in coordinates 1 and 2).

Agent B: Let $U_B = \operatorname{span}\{e_3, e_4\}$. Define $x\, R_B\, y$ iff $x - y \in U_B$.

Agent C: Let $U_C = \operatorname{span}\{e_1 + e_3,\ e_2 + e_4\}$. Define $x\, R_C\, y$ iff $x - y \in U_C$.

Epistemic operators. Write $K_i\varphi$ for "agent i knows φ," $EG\varphi := K_A\varphi \wedge K_B\varphi \wedge K_C\varphi$ for "everyone knows φ," and $CG\varphi$ for "common knowledge of φ among $G = \{A, B, C\}$." Common knowledge $CG\varphi$ holds at world w iff φ holds at every world reachable from w by any finite sequence of steps along $R_A \cup R_B \cup R_C$.

Component structure and tags. Let $H = U_A + U_B + U_C$ (the subspace sum, computed with XOR addition). The connected components of the undirected graph with edge set $R_A \cup R_B \cup R_C$ are precisely the cosets $x + H$ for $x \in \mathbb{F}_2^{12}$. For each component X, define its tag as the 8-bit integer formed by the last eight coordinates of any world in X:

$$\operatorname{tag}(X) := \text{binary value of } (x_5 x_6 x_7 x_8 x_9 x_{10} x_{11} x_{12}) \in \{0, 1, \ldots, 255\}.$$

This is well-defined since H acts only on the first four coordinates. We consider valuations V assigning truth values to three propositional atoms p, q, r at each world.

A valuation is *admissible* iff it satisfies two conditions:
(F1) At $w_\star$: $\mathcal{M}, w_\star \models CG(EGp \wedge EG\neg q)$.
(F2) Among all components $X \neq X_\star$ (where $X_\star$ is the component containing $w_\star$), call X *good* if for any (equivalently, every) $x \in X$:

$$\mathcal{M}, x \models CGEGr \quad \wedge \quad CGEG(p \leftrightarrow r \oplus \operatorname{parity}(\operatorname{tag}(X))),$$

where $\operatorname{parity}(t)$ is the sum of bits in the binary representation of t modulo 2, and $\oplus$ denotes XOR. Let $S = \{X \neq X_\star : X \text{ is good}\}$. Then,

$$|S| \equiv 7 \pmod{13}, \qquad \sum_{X \in S} \operatorname{tag}(X) \equiv 45 \pmod{97}.$$

Count the number of admissible valuations. Report your answer modulo 10,007.

Answer (mod 10,007): $\boxed{4{,}814}$

Evaluation: `[Gemini: 0/5]` `[GPT-5-Thinking: 0/5]`

Metadata.

- **Difficulty:** Hard
- **Subject:** Multi-Agent Epistemic Logic
- **Technique:** Linear-subspace decomposition of $\mathsf{S5}$ knowledge components.
- **Reasoning trace.** Compute $H = U_A + U_B + U_C = \mathrm{span}(e_1, e_2, e_3, e_4) \subset \mathbb{F}_2^{12} \Rightarrow 256$ components of size $16 \Rightarrow$ apply (F1): $p \equiv 1$, $q \equiv 0$ on $X_\star$; r arbitrary (2^{16} choices) $\Rightarrow$ for each other component X with tag t: r constant (2 choices), $p = r \oplus \mathrm{parity}(t)$, q free (2^{16} choices) $\Rightarrow A_t = 2 \cdot 2^{16}$, $B_t = 2^{48} - 2 \cdot 2^{16} \Rightarrow$ form generating product $F(y, z) = \prod_{t=1}^{255}(B_t + A_t\, y\, z^t)$, multiply by 2^{16} for $X_\star$, extract residue slice $m \equiv 7 \pmod{13}$, $s \equiv 45 \pmod{97} \Rightarrow$ final count $\equiv 4{,}814 \pmod{10{,}007}$.